Maximum signal-to-noise ratio detection of radar targets involves applying a matched filter to the received signal. This produces a map of reflected power as a function of delay and frequency. An unfortunate side effect of the matched filter is delay-frequency ambiguity, which is a function of the encoding of the radar pulse. The spurious off-target responses present in the matched-filtered signal are called delay-frequency sidelobes. When multiple or distributed targets are present, these sidelobes clutter the filtered signal and make detection and interpretation difficult.

**Radar Model**

- For a coded transmitted pulse \( s \) and a received signal \( y \), the matched filter is given by
  \[
  x[n, p] = \sum_{m} s^*[m - p]e^{-2\pi inm/Ny[n,m]},
  \]
  \( x = A^*(y) \).
- Our radar model is the adjoint operation given by
  \[
  y[m] = \sum_{p=1}^{P-1} s[m - p] \left( \sum_{n=1}^{N-1} e^{-2\pi inm/N} h[n, p] \right),
  \]
  \( y = A(h) \),
  where \( h \) is the target reflectivity as a function of frequency and delay.

**Solution Method**

- With the radar measurements modeled by
  \[
  y = A(h) + n,
  \]
  where \( n \) is complex Gaussian noise, we seek a solution by solving the \( l_1 \)-regularized least-squares problem
  \[
  \arg \min_{h} \left( \|y - A(h)\|_2 + \lambda \|h\|_1 \right)
  \]
  with the parameter \( \lambda \) set according to the noise level.
- The problem is convex and easily solved by applying the proximal gradient descent algorithm, resulting in a solution procedure called iterative soft thresholding.

**Results Summary**

- Sidelobe-free waveform inversion is performed by adding the sparse solution provided by the radar model to the unmodeled noise.
- Inversion results primarily differ because the signal generator and transmitter alter each code to varying amounts (an uncoded pulse is easier to produce) and not because of the code’s suitability.

**Conclusions**

- Sparse decomposition effectively removes delay-frequency sidelobes from meteor signals, revealing “blobular” nature of non-specular trails.
- Inversion technique works with coded and uncoded radar pulses and is applicable to any sparse target.
- Quality of sidelobe-removal depends on the similarity between the actual transmitted pulse (subject to signal-generator and bandwidth constraints) and the ideal modeled pulse.