A Compressed Sensing Approach to Observing Distributed Radar Targets

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Abstract

Compressed sensing, a method which relies on sparsity to reconstruct signals with relatively few measurements, has the potential to greatly improve observation of distributed radar targets. We extend the theoretical work of others by investigating the practical problems of implementing this approach for distributed targets, first examining a discrete linear radar model suitable for compressed sensing and then discussing an example of this technique used on existing data. Potential benefits include higher possible range resolution, complete filtering of noise without sidelobes or artifacts, and the ability to identify different Doppler shifts within the same range window of a single pulse.

1 Introduction

Compressed sensing (see [1] for a good introduction) is a new data acquisition and processing technique that leverages sparsity in the signal being measured in order to reduce the number of measurements needed to accurately reconstruct the signal. Because radar signals are quite recognizably sparse in range and frequency, with typically few targets of interest within range, radar is a natural target for compressed sensing. For our desired use with meteor plasma measurements made with high-power large-aperture (HPLA) radars, we require an approach that accounts for distributed targets and allows for high resolution imaging. Compressed sensing provides these benefits and more.

Discrete linear models for radar [2] and communication channels [3] have been used with compressed sensing previously. With the goal of high resolution radar, [2] investigates the use of Alltop sequences as compressed sensing radar waveforms. For their model, they find that range and Doppler frequency resolution depend on the inverse of pulse waveform bandwidth and total sampling time, respectively. They also prove an upper bound on the target sparsity s for which solution is guaranteed with high probability and provide simulation results that indicate that the proven bound can be relaxed to $s \leq m/(2\log m)$, where m is the number of measurements. The development and results of [3] proceed in much the same manner, except for the use of spread spectrum waveforms and the application to communication channels. Both prior works provide a good foundation for using compressed sensing with radar from a theoretical perspective. What they lack are answers to more practical questions: How does the discrete model, essentially assuming point targets at very specific ranges and Doppler shifts, relate to a continuous radar model that allows distributed targets at arbitrary locations in the delay-Doppler space? How well does the technique work on real data which inevitably includes effects not present in the model? These are the questions that we address in this paper.

Our development of a radar compressed sensing method begins with the presentation of a discrete linear radar model. By examining this, we find that solving using the discrete model gives an approximate lower bound on the total target reflectivity contained in a delay-Doppler window. The resolution of this window is determined by the pulse waveform bandwidth and the choice of Doppler discretization, the latter being limited only by the number of measurements through a compressed sensing solution condition. We conclude by applying the method to ionospheric plasma data taken with the Poker Flat Incoherent Scatter Radar and find that the solution agrees with that of a matched filter, validating the compressed sensing approach in a practical setting.

2 Discrete Linear Radar Model

In order to use compressed sensing, we need a discrete linear model describing the radar that also meets certain sparsity and incoherence requirements that ensure reconstruction with high probability. Our radar model is given by

$$y_q = \sum_{k=1}^{rm} s_{rq-k+1} \left(\sum_{p=0}^{n-1} e^{\frac{2\pi i pq}{n}} h_{p,k} \right) \tag{1}$$

where y_q for $q = 1, \ldots, m$ is the sampled received signal, m is the number of samples, s_k for $k = 1, \ldots, b$ is a complex sequence giving the discrete phase shift baseband modulation signal, b is the number of bauds in the modulation signal,

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r is the ratio of baud length to sampling period, n is the number of frequencies included in the discretization, and $h_{p,k}$ are the target reflectivity coefficients. There is an exact expression for the coefficients, but it additionally depends on q. We require that the reflectivity coefficients merely approximate this exact expression, as in

$$h_{p,k} \approx h_{p,k,q} = \int_{(k-1)\tau_b}^{k\tau_b} \int_{(p-1)\Delta f}^{p\Delta f} e^{2\pi i q \tau_s (f_d - p\Delta f)} e^{-2\pi i (f_0 + f_d) t_d} h(t_d, f_d) df_d dt_d \tag{2}$$

where τ_b is the baud length, Δf is the frequency discretization increment (equal to the receiver bandwidth divided by n), τ_s is the sampling period, f_0 is the transmission frequency, t_d is the time delay, f_d is the Doppler frequency shift, and $h(t_d, f_d)$ is the positive real reflectivity function. The approximation will be good if either the reflectivity function is nonzero only very close to the specific frequencies given by $p\Delta f$ for $p = -n/2 + 1, \ldots, n/2$, or if the first complex exponential term stays close to 1 over the limits of the Doppler frequency integral, $e^{2\pi i q \tau_s \Delta f} \approx 1$. The latter is true if we have $q\tau_s \Delta f \ll 1$ for all $q = 1, \ldots, m$ or, in terms of the number of frequency steps, $n \gg m$.

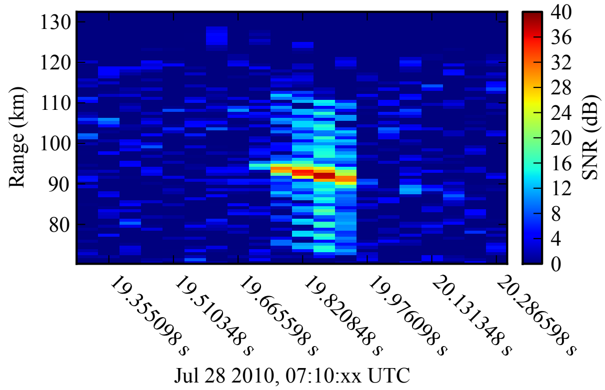
Radar targets, even the distributed ones, are typically localized enough in the delay-Doppler space that we can expect few of $h_{p,k}$ to be nonzero. Thus, this model readily meets the compressed sensing requirement of sparsity, in this case with respect to the standard basis for the delay-Doppler space. One question that remains is whether these measurements qualify as incoherent so that the guarantees of compressed sensing can be invoked. Of course, the answer to this question depends on the modulation sequence s_q. In practice, we have used this model successfully with random binary sequences ($s_q \in \{1, -1\}$) and the Barker-13 code. Alltop sequences were proven to result in incoherent measurements with a similar model [2]. Numerous papers discuss the incoherence properties of convolution or Toeplitz sensing matrices formed from random sequences [4, 5, 6] or chirp sequences [6], a structure seen in the convolution portion of our model, while [7] discusses sensing with Gabor frames, a feature which arises from the DFT portion of our model. So although we have not proven incoherence for any class of measurements made by this model, we nevertheless expect that many classes of modulation sequences will admit a compressed sensing solution.

Leaving the compressed sensing concerns behind, there is still the question of how well this discrete model describes a distributed target. One simple observation to make from equation (2) is the following:

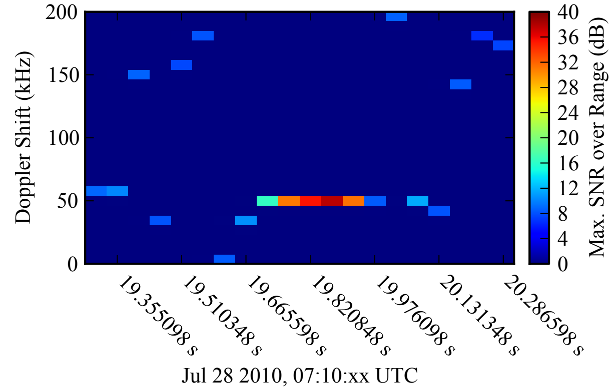
$$\begin{aligned} |h_{p,k}| &\approx \left| \int_{(k-1)\tau_b}^{k\tau_b} \int_{(p-1)\Delta f}^{p\Delta f} e^{2\pi i q \tau_s (f_d - p\Delta f)} e^{-2\pi i (f_0 + f_d) t_d} h(t_d, f_d) df_d dt_d \right| \\ &\leq \int_{(k-1)\tau_b}^{k\tau_b} \int_{(p-1)\Delta f}^{p\Delta f} h(t_d, f_d) df_d dt_d. \end{aligned} \tag{3}$$

Therefore, the absolute value of $h_{p,k}$ gives us an approximate lower bound on the total target reflectivity contained in the time delay and Doppler frequency shift window given by $[(k-1)\tau_b,\ k\tau_b] \times [(p-1)\Delta f,\ p\Delta f]$. While this may not be a very tight lower bound (in fact one can imagine a distributed target with high total reflectivity that results in $h_{p,k} = 0$ because of destructive cancellations caused by the complex exponentials), it at least allows us some way of relating the measured discrete signals to a distributed reflectivity profile.

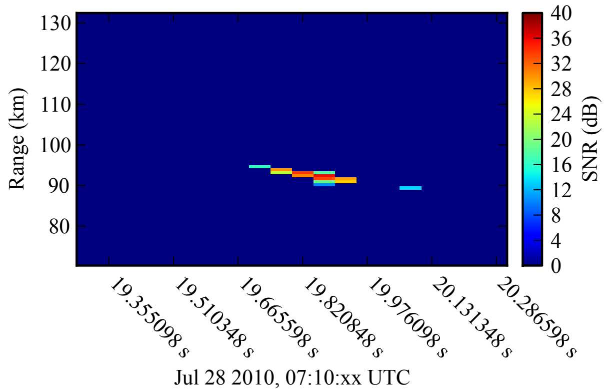
Another thing equation (2) tells us is what the discrete model cannot do. The first complex exponential in equation (2) varies in phase by an amount of $2\pi q \tau_s \Delta f$ over the Doppler shift integral, while the second complex exponential varies in phase by an amount approximately equal to $2\pi f_0 t_b$ over the time delay integral, where the approximation comes from an assumption that $f_d \ll f_0$. One way for $|h_{p,k}|$ to be a good estimate of the target's total reflectivity within the delay-Doppler window is if both of these phase shifts are small. We already noted that this is true for the first complex exponential if $n \gg m$. For the second complex exponential, this small phase shift requirement becomes $t_b \ll 1/f_0$. Making $n \gg m$ is achievable, but potentially puts a strain on the number of measurements required in order to ensure that compressed sensing arrives at the correct result. The requirement on the baud length, however, is impossible to achieve. Since $1/t_b$ is limited by the radar's transmission bandwidth, the requirement demands that the transmission bandwidth be much greater than the transmission frequency! Thus, the only realistic way that $|h_{p,k}|$ is a good estimate of the target's total reflectivity within the delay-Doppler window is if the reflectivity function is highly localized, which is exactly the case for point targets. Nevertheless, the model remains useful even for distributed targets because of the lower bound of equation (3) and the prospects of using it in conjunction with compressed sensing techniques.



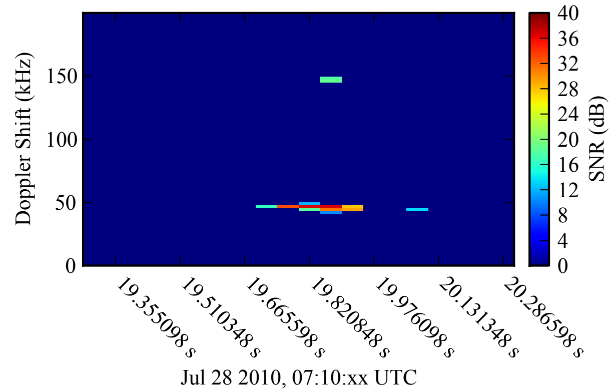
(a) Range dependence of SNR from matched filter



(b) Doppler dependence of SNR from matched filter



(c) Range dependence of SNR from compressed sensing



(d) Doppler dependence of SNR from compr. sensing

Figure 1: Decoding of Strong Meteor Head Echo

3 Example

Our example is a particularly strong meteor head echo taken from two hours of data obtained by the Poker Flat Incoherent Scatter Radar on July 28, 2010. The measurements were made using a Barker-13 code with a baud length of 10 microseconds and a sampling period of 5 microseconds. It should be stressed that these parameters are not ideal for compressed sensing, as they were chosen with matched filter processing in mind. Nevertheless, the Barker-13 waveform is a discrete phase shift code as required by our model, and it results in measurements that are sufficiently incoherent so that using compressed sensing is possible.

Depicted in Figure 1 is the SNR (signal to noise ratio) of the meteor head as a function of range and pulse time as given by the matched filter (Figure 1a) and compressed sensing (Figure 1c). Of course, the compressed sensing solution also yields the SNR as a function of Doppler frequency, and this is shown in Figure 1d. In order to arrive at the matched filter result, it is necessary to try multiple filters that have each been frequency shifted by a different amount in order to account for the Doppler shift of the returning signal. The single matched filter result is then taken to be the one that results in the highest SNR out of all of the shifted filters. This maximum SNR and the corresponding frequency shift are shown in Figure 1b for comparison with the compressed sensing result.

The first thing to note about this example is that the matched filter and compressed sensing results agree, showing approximately the same SNR for the signal at the same locations in range and Doppler frequency shift. Although it is only one example, at the very least this tells us that our approach is valid and can work on real data sets. Perhaps the second most striking takeaway from this result is the lack of noise in the compressed sensing solution, especially when compared to the range sidelobes most notably present in Figure 1a. This is a natural consequence of the compressed sensing approach, which searches for a sparse solution that falls within noise bounds. A similar effect could be achieved in the matched filter case by setting all SNRs below a certain noise threshold to zero, and this is often done in practice to separate signals from noise. The difference is that with compressed sensing, it is not possible to misidentify filtering artifacts (such as range sidelobes) as signal. The third important observation is that the compressed sensing solution associates multiple Doppler frequency shifts with each pulse, whereas the matched filter is limited to one. For a meteor head echo, this can tell us that different portions of the plasma are moving at different speeds, resulting in a range of Doppler frequency shifts that we can resolve with compressed sensing. This is perhaps the biggest immediate benefit that can be had by employing our technique.

4 Conclusion

Compressed sensing provides an exciting new way to look at radar signals. The radar model that we presented provides insight into the relationship between the compressed sensing solution and a distributed target's reflectivity in the form of an approximate lower bound of the total reflectivity in a time delay and Doppler shift window. Our model is very similar to the previous models by [2] and [3] for which compressed sensing has been explored theoretically, lending mathematical support to our procedure. From our example of a meteor head echo using data from the Poker Flat Incoherent Scatter Radar, we know that the compressed sensing procedure works and can provide new insight compared to current techniques.

One of the many benefits of using compressed sensing for radar is the ability to discern multiple Doppler shifts not only within the same pulse but also within the same range window. For meteor studies with HPLA radars, this ability is vital to elucidating the complex processes present in the plasma. In addition, compressed sensing provides the opportunity for higher range resolution when compared to traditional techniques since the time delay resolution of its solution is not directly limited by the sampling period. The high range resolution can be achieved with high sensitivity since compressed sensing waveforms allow one to choose baud length and pulse length arbitrarily and independently. A natural consequence of compressed sensing's search for a sparse solution is that the result identifies the location of signal only, filtering out all noise without sidelobes or other artifacts.

Acknowledgments

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